Logic, Reasoning, and Revision

Abstract

The traditional connection between logic and reasoning has been under pressure ever since Gilbert Harman attacked the received view that logic yields norms for what we should believe. In this paper I first place Harman's challenge in the broader context of the dialectic between logical revisionists like Bob Meyer and sceptics about the role of logic in reasoning like Harman. I then develop a formal model based on contemporary epistemic and doxastic logic in which the relation between logic and norms for belief can be captured.

INTRODUCTION

The canons of classical deductive logic provide, at least according to a widespread but presumably naive view, general as well as infallible norms for reasoning. Obviously, few instances of actual reasoning have such properties, and it is therefore not surprising that the naive view has been challenged in many ways. Four kinds of challenges are relevant to the question of where the naive view goes wrong, but each of these challenges is also interesting because it allows us to focus on a particular aspect of the relation between deductive logic, reasoning, and logical revision. Challenges to the naive view that classical deductive logic directly yields norms for reasoning come in two sorts. Two of them are straightforwardly revisionist; they claim that the consequence relation of classical logic is the culprit. The remaining two directly question the naive view about the normative role of logic for reasoning; they do not think that the philosophical notion of entailment (however conceived) is as relevant to reasoning as has traditionally been assumed.

I provide a novel account of how logic constrains our beliefs. This account replaces Harman's belief-box metaphor by the Stalnakerian metaphor of *logical space*, and characterises beliefs as well as defeasible inference forms as types of soft information that are associated with, respectively, agents and a logical space. The underlying formalism is inspired by contemporary epistemic and doxastic logic, and is meant to be agnostic with respect to the nature of defeasible inferences.[1]

[1]Russell (2006) is, as far as I know, the earliest comparison of contemporary dynamic logics of belief with Harman's scepticism about how logic may be related to norms for belief(-change).

Overview of the paper This paper consists of two complementary parts; with a first part focused on Harman's challenge to the received view that logic could be a theory of reasoning and what this means for logical revision, and a second part in which a model for the relation between logic and reasoning is outlined, and used as a partial response to Harman's challenge.

In the first part, the leading theme is the dialectic between logical revisionists, and sceptics about the role of logic in reasoning. A first section introduces the four protagonists of our discussion: the simple revisionist (modelled after Bob Meyer), the sophisticated revisionist (a generic character that defends the adoption of a non-monotonic logic), the basic sceptic (modelled after Gilbert Harman), and the critical sceptic (a generic character that draws attention to the gap between logical principles and norms for belief). In section 2 I take on *basic scepticism*: I summarise Harman's critique on the traditional connection between deductive logic and norms for reasoning, explain why the resulting challenge to the traditional view on how logic yields norms for reasoning is hard to meet, and finally discuss what it could mean for logic to be *specially* relevant to reasoning. The upshot of this section is to find a balance between taking Harman's scepticism seriously, and leaving some room to establish a credible connection between logic and reasoning. In Section 3 I use a problem for the simple revisionist, namely the possibility of revisionary slides, to discuss the dialectic between the four positions I introduced earlier.

For the second part, I outline a model for a logical space that can be used to represent monotonic consequence-relations as strict modalities, and non-monotonic consequence-relations as variable modalities (§4). This approach generalises the distinction between knowledge-like *hard information* and belief-like *soft information* from contemporary epistemic logic to our reasoning about logical information. The proposed model is subsequently (§5) put to work by showing how it can be used to conceptualise logical revision, put in perspective by explaining how it relates to a recent debate between Beall and Priest over the value of non-monotonic logic, and further developed by defining different types of logically constrained belief. The final section in this part (§6) returns to the problem of revisionary slides that was introduced in the first part, shows how it can clarify the positions of the sophisticated revisionist and the critical sceptic, and in what sense it can be seen as an answer to Harman's challenge.

1 THE PROTAGONISTS

The ***simple revisionist*** has it that since logic should infallibly guide our inferences, the fact that classical logic warrants inferences that are epistemically unacceptable—*ex falso quodlibet*, to name just the more obvious one—shows that classical logic is in need of revision. Revision, in this case,

means that some classically valid arguments need to be rejected. Prototypical simple revisionists are most often found among relevant and other paraconsistent logicians.[2] The kind of argument given in Bob Meyer's *Entailment* (Meyer 1971) is part of the folklore and is at least implicitly used to motivate many non-classical systems.

> [I]t is an empirical fact that (1) some people sometimes are committed to some contradictory beliefs. And again, what else is logic for if it is not the case that (2) a man committed to certain beliefs is committed as well to their logical consequences? Friends, it is downright odd, silly, and ridiculous that on classical logical terrain (1) and (2) cannot be held together, except on pain of maintaining that some people sometimes are committed to absolutely everything.
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> (Meyer 1971, 814)

Up to a certain point, simple revisionism is good enough to motivate what logicians do, but it is not good enough for epistemologists who want to understand the norms logic might or might not impose on our beliefs. As I see it, simple revisionists are right when they believe that there are good reasons for revising classical logic, and that some such revisions essentially involve dropping some classically valid arguments—semantic paradoxes are the prime example (see e.g. Field (2008, 14–17) on such reasons to revise classical logic). Yet, they could be blamed for overemphasising the epistemic role, and thus the unacceptable consequences, of classical logic. Revisionary arguments of this simple kind[3] should primarily focus on how logic relates to argument, perhaps also on semantics, but much less on adopting new beliefs that are implied by previous beliefs.[4]

Most ***sophisticated revisionists*** believe there is something fundamentally wrong with classical logic, but disagree with the simple revisionist's way of resolving this issue. Dropping some argument-forms while leaving structural properties of the consequence-relation like monotonicity, identity and transitivity intact,[5] cannot be the right answer. We also need a structural revision of logic. This line of thought is shared by proponents of

[2]In general, logical revisionism isn't limited to paraconsistent enterprises. The intuitionist case for revising classical logic, the many non-classical approaches to the semantic paradoxes, and the sorites are all revisionary projects. Yet, when we focus on the connection between logic and belief, paraconsistency plays a central role.

[3]This is a bit misleading, since the intuitionistic case for logical revisionism, which belongs to what I call *simple revisionism*, has become quite sophisticated (e.g. Wright 1994). In addition, epistemology can also be relevant outside the discussion of how logic constrains our reasoning, but it is not in that sense that I'm using the term here. What I want to emphasise by calling such revisionism simple is that it proposes revisions based on the rejection of classically valid argument-forms.

[4]I ignore for now the gap between "being committed to" a certain belief, and "adopting" a new belief.

[5]A more precise formulation would be that simple revisionists leave the structural properties of the *external* consequence relation intact. Many substructural logics, and especially

non-monotonic systems (Batens 1997, Horty 2007, 2012, Pollock 1995), but unlike for the simple revisionist, this line of thought does not depend on a common revisionary argument. Whereas Batens (1997) and Priest (2006*b*, Chapt. 16) are clearly concerned with the problem of paraconsistent logical revisionism, other authors like Gabbay & Woods (2008) focus more on the role of resource-bounded reasoning.[6] As a consequence, what I call sophisticated revisionism is hardly a unified position. The label refers in the first place to the view that since most of our reasoning is defeasible because it equally exploits the presence and absence of information, the logical systems we use to describe this reasoning should be based on defeasible or nonmonotonic relations (conditionals, but also consequence-relations). The mentioned lack of unity is in line with the diversity in non-monotonic systems, but also reflects the comparative lack of philosophical foundation of these systems.[7] As a consequence, it is much harder to say what exactly is right or wrong with sophisticated revisionism. One aim of this paper is to find out if there is a coherent argument behind this form of revisionism; both with regard to how logic is to be revised and with respect to our reasons for doing so.

Next, the ***basic sceptic*** professes that we should observe the difference between the logical notion of implication and the methodological notion of inference. The view that implication and inference are separate notions is championed by Gilbert Harman, who questions the naive view, and therefore also the simple revisionist's argument (Harman 1986). If valid implications are not primarily norms for inference, their failure to provide all and only rational norms for inference[8] cannot be a reason to revise the canons of logic. The main virtue of this kind of scepticism is the distinction between logical notions like implication and validity and methodological notions like inference. It is important to note that the value of this distinction is largely independent from any other consideration about the nature of logic or its normative role. What Harman emphasises is that merely stating a logical principle does not suffice to say anything about what one should believe. The basic sceptic's overtly negative conclusion is, even if one accepts the distinction between implication and inference, more controversial: "[T]here is no clearly significant way in which *logic* is

the relevant logics favoured by Meyer are obtained by restricting the structural rules of a calculus. This leads to a revision of the structural properties of the internal consequence relation, but not necessarily to a revision of the external consequence relation. A detailed overview of the distinction can be found in Mares & Paoli (2014). Here, we need not bother about the precise origin of the distinction, and only need to note that when we talk about what follows from our beliefs, or about the consequences of a given theory, we appeal to an external consequence relation.

[6] On the connection between nonmonotonic reasoning and realistic human reasoning, see for instance Dutilh Novaes (2012, 16.2) and Strasser & Antonelli (2014, §4).

[7] But see Horty (2012) for a specific discussion on how non-monotonic logic is related to reasons.

[8] In Harman's view, inference is tied to change in belief, so norms for inference are by definition norms for belief revision.

specially relevant to reasoning." (Harman 1986, 20). Thus, it seems that if the basic sceptic is right, we might need an error-theory to explain the historical connections between logic, argumentation and method. Putting matters this way is somewhat misleading, and a more fruitful way to frame Harman's rejection of this historical connection is as a disagreement about how logic is *specially relevant* to reasoning, about how it leads to a norm that is different or more constitutive than other extra-logical norms.

Critical sceptics are largely in agreement with Harman's point of view, but add novel insights as well.[9] In particular, there is the suggestion due to John MacFarlane that logic can only be normative for thought if there is a bridge principle that connects valid entailments to deontic statements about doxastic states, where the latter are permissions or obligations to believe certain propositions (MacFarlane 2004). Unlike Harman—who seems to argue that there are no such bridge principles—the critical sceptic points to the many difficulties a plausible bridge principle ought to cope with, and above all emphasises that revisionary arguments invariably depend on unarticulated but highly implausible bridge principles. As shown by Fitelson with regard to Goodman's Riddle of Induction (Fitelson 2008), such bridge principles are relevant to all sorts of formal epistemology, and several revisionary arguments can be shown to depend on equally implausible bridges between logical and epistemic relations. Recently Steinberger (2014+) further elaborated on Fitelson's thesis that no plausible bridge principle could be used in a revisionary argument, and argued against MacFarlane's more optimistic suggestion that with the right bridge-principles we might be able to settle debates about validity by referring to normative principles for thought.

A further addition to the critical sceptic's position is due to Titelbaum (2008, 2013, §2.1), who explains that we should not think of only two relata that could be bridged, but of three: (a) formal systems, (b) philosophical notions, and (c) sets of norms. As before, the upshot is that whenever we move from one to the other, we make use of a bridge principle. By adding a third relatum, we accept that doxastic norms can be modelled directly; without having to refer to a philosophical concept like logical entailment and/or implication as some sort of go-between. If we choose to do so, we can simply by-pass Harman's Challenge. As a result, simple revisionary arguments no longer lead to the conclusion that we need to revise our logic (understood as either the philosophical notion of entailment, or as our formal theory thereof).[10] Crucially, this approach doesn't imply that

[9] Whereas the distinction between simple and sophisticated revisionists is sharp (and presumably widely agreed upon), the distinction between basic and critical sceptics is a bit more artificial. The way I want to use the distinction is nevertheless unproblematic. I consider the latter as an elaboration of the former, and believe that both views only differ in their emphasis and final conclusions.

[10] This distinction bears on the question of whether logical revision concerns our theory of logic, or its subject-matter, namely Logic itself. The distinction between logic—the theory—and Logic—what logic is about—is due to Priest (see e.g. Priest 2006*a*).

entailment-relations are irrelevant to reasoning, but only that the question doesn't need to be answered if all we want is a formal model of (deductive) reasoning.

2 HARMAN'S CHALLENGE

Apart from being famous, Harman's contention that logic isn't specially relevant to reasoning has the following distinctive features.

First, it is a multi-faceted objection to the received view that the laws of logic are (or provide) general as well as infallible or exception-free rules for reasoning, rather than a focused attack on this orthodox view. In outline, Harman contends that logic cannot be a norm for reasoning because (a) implication and reasoning belong to different categories; (b) logic is cumulative, but reasoned change in view is not; (c) rules of argument are indifferent between adding a new belief, and giving up a prior belief; (d) logical principles are exceptionless, but instructions for changing one's beliefs can have exceptions; (e) inconsistent beliefs imply anything, but that doesn't force one to either adopt a trivial belief-set or to revise one's inconsistent beliefs; (f) we need to avoid clutter (irrelevant beliefs), but our beliefs imply many such irrelevant beliefs; (g) principles of implication do not say something precise about reasoning. I shall henceforth refer to the totality of these objections as "Harman's Challenge".

Second, meeting this challenge isn't just hard because of the wide range of objections that need to be met, but also because we do not merely need to show that logic is relevant for reasoning, but actually need to show that logic is *specially* relevant for reasoning; a requirement that can be illustrated by the following question: "Or should we think of logic as having a special role in reasoning, a role that is not simply a consequence of its wider application?" (Harman 1984, 107).

♦ ♦ ♦

Because Harman identifies several problems within the naive view that classical logic supplies infallible norms for reasoning, any explanation of why the traditional connection between logic and reasoning isn't mistaken will have to include solutions for each of the issues pointed out by Harman (i.e. either rebuttals of the objections or some other way to deal with the objections). When put in the terminology of the *critical sceptic*, any attempt to formulate a bridge-principle that relates logical principles to norms of reasoning will have to deal with such disparate issues as *clutter avoidance* and *inconsistency tolerance*. Quite obviously, any bridge-principle that meets these demands is exceedingly complex (for it has to take into account several exceptions), and runs the risk of only establishing a very indirect connection between logic and reasoning (because handling exceptions means

introducing extra-logical considerations). Such an indirect connection, so the thought goes, may perhaps show that logic is somehow relevant to reasoning, but would never fully meet Harman's Challenge. Conversely, partial connections may establish a more direct connection, but only by leaving a gap elsewhere.

To a first approximation, a bridge-principle is partial when it doesn't properly deal with all the counterexamples, or when it only relates logical principles to some intermediate notion (for instance, constraints on belief-states rather than instructions on how to change one's beliefs). In the former case, we may hope to obtain a direct connection between logic and reasoning, but that connection would presumably only hold in a suitably restricted domain. In the latter case, an additional connection between the intermediate notion and proper instructions to change one's beliefs will have to be posited. In each case, the restrictions we may need to impose on the domain,[11] or the additional connection one would have to posit could threaten the intended privileged connection between logic and reasoning.[12] As such, each attempt to deal with all facets of Harman's Challenge will run into troubles with the second demand of showing that logic is indeed *specially* relevant for reasoning.

In view of the above discussion we may be tempted to conclude that it is indeed impossible to meet Harman's Challenge because it imposes two incompatible requirements. We need a sufficiently direct connection, but at the same time we also need to cope with a wide range of exceptions. Clearly, these two requirements pull in opposite directions, and this may suggest that the constraints imposed by Harman's Challenge are too restrictive. Of course, the challenge in question makes sense when it is directed at the naive view, but when it is directed at any attempt to formulate logical constraints on belief and belief-change it becomes misleading. The role of the word "special" is crucial in that respect, for it suggests that only the naive view yields a sufficiently direct connection between logic and reasoning.

Consider, as a point of comparison, the standard paraconsistent view that, as a requirement on theories, consistency isn't any different from other virtuous properties of theories like elegance, explanatory power, simplicity, parsimony and informativeness. All else being equal, consistent theories are preferable to inconsistent theories, but *ad hoc* consistent theories are often worse than less *ad hoc* inconsistent theories. As a result, consistency doesn't always take precedence over, say, explanatory power. This

[11] Field's use of the notions of "recognised implications" and "recognised inconsistencies" (Field 2009) can be seen as such a restriction (see Harman 2009).

[12] An additional version of the problem is this. Assume that for each type of counter-example we have a partial bridge-principle that satisfactorily deals with all instances of that counter-example. We have, so to say, several bridge-principles that operate independently of each other. Even if each of these partial principles reveals a direct (though not exception-free) connection, there's no guarantee that by combining these principles (if feasible at all) we would obtain a direct connection as well.

can happen synchronically, when we need to decide between a logically cogent theory with poor explanatory power and a theory that operates on an inconsistent basis while being more explanatory, but also diachronically, for instance when we ponder the revision of an inconsistent yet highly explanatory theory. Presumably, this means that consistency isn't a special requirement on theories. In Priest's view, this is mirrored by the fact that he doesn't take consistency to be a hard logical requirement on theories, for logical requirements do always take precedence over the previously mentioned properties of explanatory power, simplicity, *etc.* While this doesn't yet tell us something about logical constraints on belief, it at least shows that we can make sense of logic as a *special* requirement on theories. Analogously, we can take the requirement for a special role for logic in reasoning to mean that logic should impose constraints that do not need to be balanced with other, non-logical, requirements.

Alternatively, and this is the path I shall pursue, we can take the *specialness* requirement to mean that even though logical requirements may need to give way to non-logical requirements, they impose a different kind of requirement—a requirement with features that are traditionally associated with logic like *formality* and *necessity*. In other words, I will argue that while some of our logical information is belief-like rather than knowledge-like (it is so-called *soft* information and thus in a sense open to revision), it is also a special kind of soft information, and requires a special approach to theory-revision.

3 REVISIONARY SLIDES

Being serious, but also realistic about the normative role of logic in reasoning commits us to either sophisticated revisionism or to critical scepticism,[13] but it doesn't make the more crude proposals irrelevant to this issue. In fact, since sophisticated revisionism as well as critical scepticism is open to the suggestion that all we need is a formal model (which, as I pointed out earlier, fails to address Harman's challenge), even more is to be gained from a closer look at the dialectic between all four positions.

The main problem for basic revisionism is that it seems to trigger revisionary slides:[14] Once one accepts the cogency of revisionary arguments, one has to accept many of its instances. If such slides are forceful, the basic revisionist can only maintain the naive view on the normative role of logic in reasoning relative to extremely impaired logical consequence relations.

[13] As correctly pointed out by a referee, and further illustrated in this section, simple revisionists might simply be too serious about the normative role of logic . Hence, the further qualification that we should also be realistic about the role of logic.

[14] This description is strongly influenced by Batens (1997). A similar idea can be found in the literature on logical pluralism, where the idea that validity in all logics has an empty or close to empty extension is seen as an argument for recognising multiple logics as good accounts of logical consequence (Beall & Restall 2006, Bueno & Shalkowski 2009).

This, arguably, is hardly the outcome the basic revisionist could have had in mind. To maintain that the naive view isn't vacuously true, the basic revisionist needs a way to resist the conclusion of revisionary slides.

To begin with, we need to show that revisionary slides are indeed possible.[15] Let's start with the following schematic revisionary argument (with "P" and "Q" predicates):

1. One can be rationally committed to a set of beliefs that is P.
2. One is committed to the logical consequences of one's beliefs.
3. A set of formulae that is P entails a set of formulae that is Q.
4. Being committed to a set of beliefs that is P means being committed to a set of beliefs that is Q.

Hence, as soon as being committed to a set of beliefs that is Q is considered irrational or otherwise epistemically unacceptable, the naive view expressed in (2) can only be maintained by rejecting either (1) or (3). Assuming (1) therefore forces one to dismiss (3), which is to follow the revisionary path. The traditional revisionary argument is obtained from the schematic version by substituting "inconsistent" for "P" and "trivial" for "Q". To construe a plausible revisionary slide, this approach remains too coarse, and has to be complemented with a more fine-grained approach to inconsistency.

To illustrate the general idea, we consider three instances of the schematic argument:

First instance: inconsistency and triviality. Call a set of formulae negation-inconsistent whenever it entails both a formula and its negation. The first step of our revisionary slide is then obtained by substituting negation-inconsistent for "P" and trivial for "Q" in the generic revisionary argument. To avoid the collapse of negation-inconsistent beliefs into trivial beliefs, the first revisionary move leads to a paraconsistent logic. Presumably, the process of revising one's logic obeys principles similar to those for ordinary belief-revision.[16] The latter includes a principle of minimal mutilation, which makes it quite plausible that our first revisionary move should lead to a maximal paraconsistent logic. That is, a logic which has no paraconsistent extensions, and has classical logic as its sole non-trivial extension. Several well-known paraconsistent logics fit this description: Priest's **LP** (Priest

[15] In general, only the law of identity is considered to be relatively stable in the face of revisionary arguments, but even that principle could be rejected for being circular (MacFarlane (2004), following a suggestion from Broome). Even though logical systems which exclude circular arguments have indeed been conceived in the relevantist tradition, I leave it open whether revisionary slides really have to lead to the empty logic rather than to other uninterestingly weak systems.

[16] This is consistent with the view, defended by Priest (2006*a*), that when we revise our logic, we revise our theory of what the correct logic might be rather than the correct logic itself.

1979) and Batens' **CLuNs** (Batens 1980, Batens & De Clercq 2004) are the most obvious ones. The only difference is that the latter has a detachable implication; for present purposes, I shall adopt that one.[17] A distinctive trait of such logics is that they still allow inconsistencies to *spread*. For instance, the negation-inconsistent set $\{p, q, \neg(p \wedge q)\}$ entails the following disjunction of further contradictions $(p \wedge \neg p) \vee (q \wedge \neg q)$. This feature is the starting point for a second instance.

Second instance: spreading contradictions. Call a set of formulae maximally negation-inconsistent whenever for each negation-inconsistent formula of the form $A \wedge \neg A$ it contains, it also contains a formula with contradictions at the atomic level. For instance, where A has the form $p \wedge q$, a maximally negation-inconsistent set that contains $A \wedge \neg A$ will also contain $(p \wedge \neg p) \vee (q \wedge \neg q)$. In maximally paraconsistent logics this *spreading* of inconsistencies is typically ensured by the *De Morgan* laws. Using the distinction between negation-inconsistency and maximal negation-inconsistency, a second step from our revisionary slide is obtained by substituting negation-inconsistent for "P" and maximally negation-inconsistent for "Q".

Admittedly, the spreading of inconsistencies isn't quite as bad as the collapse into triviality,[18] but the logical revision it seems to commend isn't unnatural given the prior acceptance of a paraconsistent logic. Since the previous choice for a maximal paraconsistent logic was guided by a principle of minimal mutilation, the move to a weaker paraconsistent logic isn't too costly either. In this case, fewer logics fit the description: Batens' **CLuN** (Batens 1980), which is obtained by adding excluded middle to the positive fragment of classical logic, is the prime example. Because it still allows for all positive inferences, most reasoning that merely involves the combination of premises as well as the application of *modus ponens* can still be carried out within this system. This ability to combine premises is the target of a third possible instance.

Third instance: strong and weak inconsistency. Call a set of formulae *strongly inconsistent* if it contains two formulae A and B such that adding $A \wedge B$ to that set suffices to make it negation-inconsistent. Call a set of formulae *weakly inconsistent* if it contains $n > 2$ formulae A_i and m formulae B_j such that adding the conjunction of all A_i's and the conjunction of all B_j's to that set would suffice to make it *strongly inconsistent*. Following Klein's (1985) use of this distinction, we can construct a final revisionary argument. To simplify matters, consider a set that is weakly inconsistent

[17] Since as consequence-relations neither of these is included in the other (the implication in **LP** contraposes, but doesn't detach whereas the implication in **CLuNs** detaches, but doesn't contrapose.), they can both be maximally paraconsistent.

[18] In many cases the multiplication of inconsistencies makes their later removal harder.

because it contains the formulae $p_1, p_2, \ldots, p_n, \neg(p_1 \wedge p_2 \wedge \ldots \wedge p_n)$. As Klein argues, if $\neg(p_1 \wedge p_2 \wedge \ldots \wedge p_n)$ is a so-called preface-proposition (at least one p_i I believe to be true is actually false), then it may be rational to have weakly inconsistent beliefs. Yet, even though weakly inconsistent beliefs may be rational, it doesn't necessarily follow that holding strongly inconsistent beliefs is also rational. As soon as we grant Klein's point, we obtain a new revisionary argument by substituting weakly inconsistent for "P" and strongly inconsistent for "Q" in the generic revisionary argument. Plausibly, the outcome of this last revision leads to a restriction or even rejection of the positive inference of adjunction that allows us to close finite sets of formulae under conjunction.

These three examples suffice to illustrate the general idea of a revisionary slide, for if even conjunction-principles can be challenged, it is hard to see which logical principle could be wholly immune to such challenges. We could for instance also blame the rule of addition that allows us to weaken any formula that is already accepted by adding as many irrelevant disjuncts as we want.

A revisionary slide can be seen as a parallel or sequential presentation of the more informal, and widely endorsed claim, that if one accepts that logic imposes strict norms on what we may and should believe, virtually any logical principle can be challenged.[19]

Since revisionary slides potentially make the claim that one is committed to the logical consequences of one's beliefs vacuously true, the simple revisionist has no choice but to try to stop the slide. Assuming that also the sophisticated revisionist and the sceptics need to say something about revisionary arguments, revisionary slides have a wider impact. The specific problem faced by the simple revisionist is that he needs to stop the slide while retaining the first step of that slide. The basic sceptic, by contrast, can simply block the first step by denying the crucial second premise that one is committed to the logical consequences of one's beliefs. Alternatively, the basic sceptic could also just accept the conclusion that one can be committed to an absurdity whenever one has inconsistent beliefs, and only deny that this would also be a reason for adding that absurdity to one's beliefs. Either way, no logical revision should be triggered.[20]

When presented as an objection to the simple revisionist that purportedly shows the superiority of the position of sceptic, the possibility of re-

[19] This is of course consistent with the claim, defended in Steinberger (2014+), that no plausible bridge principle between logic and belief would support such an argument

[20] The distinction between "adding to one's beliefs" and "merely being committed to" what follows from one's beliefs could perhaps be useful to the simple revisionist as well. Indeed, it might suffice to first deny that one should believe everything one is committed to, and then point out that while it is still a problem to be committed to an absurdity, it is admissible to be committed to further contradictions or to explicit contradictions. Of course, this would mean that even the simple revisionist could not endorse a direct connection between logic and the beliefs one should adopt (but see Milne (2009) for a discussion of how belief, commitment and logic could be related).

visionary slides seems to carry some force. However, the sheer popularity of the logic **LP** as well as the general tendency to avoid the weaker paraconsistent logics suggest that paraconsistent logicians are not particularly moved by revisionary slides. As soon as the threat of triviality is avoided, there is no urgent need to further revise logic. This tendency results, however, from a gradual (weaker logics are too be avoided) rather than from a principled reason to reject further logical revisions, and ignores in particular the epistemic benefits of localised inconsistencies in problem-solving contexts where restoring consistency is a long-term goal.

Revisionary slides also reveal a similarity between the attitudes of the sophisticated revisionist and the critical sceptic. Both agree that the moral of revisionary slides is that there are no exceptionless logically based norms for reasoning. Yet, where the sophisticated revisionist claims that this shouldn't pose a problem for the view that there are logically based norms, the critical sceptic will claim that the second premise of the revisionary argument is the real culprit. The difference between the critical sceptic and the sophisticated revisionist is that the latter argues in favour of the adoption of a logic that allows for exceptions—a non-monotonic logic—, whereas the former claims that any plausible bridge-principle should allow for exceptions—a defeasible principle. In sum, the disagreement reduces to the level at which we should accommodate exceptions. If we keep in mind the second lesson of the critical sceptics, this also means that sophisticated revisionists as well as critical sceptics may settle on the same formal system and yet disagree on what may be called logic proper. That is, they may agree on the resulting norms, on how these are modelled, but not necessarily on how these norms are related to the philosophical notion of logical consequence.[21]

The formal model outlined in the next sections can be seen as a potential common ground between the sophisticated revisionist and the critical sceptic, and can be used to explain how both positions can understand the role for logic in reasoning.

4 A FORMAL MODEL

On Harman's account, beliefs are modelled in accordance with the so-called *belief-box* metaphor. Our beliefs are sentences that are somehow stored in our brain. It is this assumption that puts clutter-avoidance high on Harman's agenda, and which makes the idea of the deductive closure of our beliefs untenable. In the model I shall lay out, and which is based on the familiar metaphor of logical space (Stalnaker 1984), no such assumptions are made. As I see it, modelling knowledge and belief in terms of what

[21] I deliberately use the more generic term "logical consequence" because the proponent of a non-monotonic logic might claim that his/her preferred account of defeasible inference is a logical consequence relation, but not an entailment relation.

is true or false in a certain sub-set of a larger logical space is just a way to think about knowledge and belief at a very high level of abstraction; a level where the actual storage of beliefs isn't taken into consideration, but where other issues regarding the connection between logic and reasoning can still be assessed. As I will argue, this level of abstraction is just fine if we're concerned with the question of failures of deductive cogency,[22] and the related issue of logical revision.

Let me make this a bit more precise. Where S is a space of possibilities, each $s \in S$ is a possible state, and s_0 is the actual state (or a model thereof), it is standard (though not undisputed, as it leads to a highly idealised model of knowledge) to say that an agent a knows that p if and only if p is true in all states in S that a cannot distinguish from the actual state. To avoid the controversy over whether this is really a definition of knowledge, I shall use the term *hard information* to denote the propositional attitude that's so defined. Concretely, where $[s_0]^{\sim}_a$ is the set of states in S that a cannot distinguish from the actual state, a's hard information will be identified with this set of states (a semantic proposition) as well as with the set of formulae that are true in all these states (a theory). Such characterisations of an agent's information obey a well-known inverse relationship principle: If $[s_0]^{\sim}_a$ is smaller, then a's theory is larger: fewer possibilities means more information.

Given this account of hard information as a formal counterpart of knowledge, we can now introduce soft information as a formal counterpart of belief. To that end, we stipulate that S isn't just partitioned into those states that a can and cannot distinguish from the actual state, but that the former, namely $[s_0]^{\sim}_a$ has an additional structure that signals which states a deems more or less plausible. One way to implement this structure is as a pre-order $\leq_a$ over $[s_0]^{\sim}_a$. For every $s, t \in [s_0]^{\sim}_a$, we read $s \leq_a t$ as saying that a deems s at least as likely as t. That is, the most likely states in a's hard information are those that are minimal according to $\leq_a$, and a's soft information can then be identified with both the set of $\leq_a$-minimal states in $[s_0]^{\sim}_a$ (again, a semantic proposition, for which I'll use the shorthand $[s_0]^{\leq}_a$ if no confusion can arise) as well as with the set of formulae that are true in that set of states (again, a theory).[23]

[22] In Christensen (2004) the term "failure of deductive cogency" refers to situations where one's beliefs are not deductively closed. In particular, it is argued that preface-like propositions can lead to *rational failures of deductive cogency*; situations in which it would be irrational to give in to deductive cogency. I rely on the same terminology of deductive and logical cogency, but use it in a much looser sense. For a recent extensive discussion of why there is no requirement of deductive consistency on full-belief, see Easwaran & Fitelson (2015).

[23] This general setting is found in many places in the literature, for instance Van Benthem (2007) and Baltag & Smets (2008). Here, I follow explicitly the latter in taking the most plausible states to be the minimal states according to the plausibility-ordering, but do not further commit to the requirement that every such ordering should be connected within an agent's hard information. For reasons that will become clear later on, some types of soft information require incomparable states; states $s, t \in [s_0]^{\sim}_a$ such that neither is more, less

Hard and soft information do not merely differ in that the latter can be false (s_0 need not be $\leq_a$-minimal in $[s_0]^{\sim}_a$), but also because soft information is open to revision. When a receives new hard information,[24] the result of updating his hard information will never lead to a decrease in hard information, but may lead to a change (rather than an increase) in soft information. This can be seen from the fact that when $[s_0]^{\sim'}_a \subseteq [s_0]^{\sim}_a$, with the former the result of updating the latter, it can still be the case that the $\leq_a$-minimal states in $[s_0]^{\sim'}_a$ do not form a sub-set of the $\leq_a$-minimal states in $[s_0]^{\sim}_a$. When, by contrast, a receives new soft information, this has no impact on a's hard information, but may lead to a change in how states are ordered.[25] Here too, there's no guarantee that the new $\leq_a$-minimal states will form a subset of the old $\leq_a$-minimal states.

By calling S a *logical* space, we emphasise that S is also a model of logical possibility. When s is a state in S, then s is logically possible, and the set of formulae that are true at s is non-trivial (and, given classical logic, consistent). If we moreover assume that every non-trivial set of formulae is true somewhere in S, it is a corollary of standard completeness-proofs that truth-preservation over S coincides with logical consequence. Jointly, the assumptions that a logical space contains all and only logical possibilities guarantee that since a's information is a subset of a logical space, it is both non-trivial and closed under logical consequence.

Standard presentations of the metaphor of logical space are based on the identification of possibility and consistency, and therefore construct a logical space that is in accordance with classical logic (Stalnaker 1984, 52–3). Non-classical extensions of this approach usually distinguish between possible states (or worlds) which, like the actual world, are consistent in the classical sense, and (logically) impossible states. This type of distinction has applications in the semantics of non-normal modal logics and many relevant logics (Priest 2001, Ch. 4, 9 and 10), but can be dispensed with in this context. Exactly like classical possible states can be identified with classical valuations, so can non-classical possible states be identified with the appropriate non-classical valuations. This holds in particular for the valuations used by the paraconsistent logics mentioned in the previous chapter. Non-classical logical spaces are exactly like classical logical spaces because they are models of logical possibility and necessity in the sense that (i) possibility coincides with logical possibility or satisfiability (the existence of a valuation that assigns a designated value to a formula or set

or equally plausible than the other.

[24] This is just shorthand for saying that a receives true information, and treats it as fully reliable. Put differently, for a to receive hard information is another way of saying that this information will affect a's hard information.

[25] Two revision-strategies are prominent in the literature, namely *radical* and *conservative* upgrades. A radical upgrade with A will change $\leq_a$ in such a way that all A states become better than all non-A states, while leaving the order among A states and among non-A states unchanged. A conservative upgrade with A will only put the best A states on top, and leave everything else unchanged. An overview of a wide-range of revision-strategies is given in Rott (2009).

of formulae), and (ii) necessity coincides with logical necessity or validity (every valuation assigns a designated value).[26] The changes to our concept of hard information are minimal. Whereas in the classical case we think of an agent's hard information in terms of what is true (and hence not false) in a set of indistinguishable states, in a typical non-classical case based on a paraconsistent logic like **LP** or **CLuN(s)** we will think of it as what is at least true (but may be false as well) in a set of indistinguishable states.

Independently of the logic that underlies a particular logical space, the resulting construction is one where an agent's information is strongly constrained by logic. The nature of this connection remains nevertheless generic. It isn't only silent regarding the actual storage of an agent's information, but it also makes abstraction of the distinctions between actual beliefs and mere commitments, or between synchronic and diachronic logical constraints on our information (see: Christensen 2004, §1.3).

♦ ♦ ♦

The above emphasis on the role of abstraction and idealisation tells us something important about Harman's challenge and about the role of bridges between logic and norms for thought (belief, reasoning, *etc.*). Take, for instance, the claim of MacFarlane (2004) that bridge-principles are connections between valid entailments and deontic statements about doxastic states. For instance, something of the form: If $A, B \vdash C$ then, if you believe that A, and believe that B, then <*some deontic modality*> believe C, where the deontic modality could be either strict (no exceptions allowed) or defeasible. Or, as in Streumer (2007), where the emphasis is on synchronic constraints:

> For all propositions $p_1, \ldots, p_n$ and q, if the conjunction of $p_1, \ldots,$ and p_n entails q, then there is a reason against a person's both believing that $p_1, \ldots,$ and p_n and believing the negation of q.
>
> (Streumer 2007, 362)

and where the reference to "reasons" acts as a defeasible doxastic modality.

Whenever such a principle includes a non-strict doxastic modality, this modality is meant to take care of the exceptions that are due to failures of deductive cogency. This view, however, glosses over the fact that (at least if we think of Harman's challenge) bridge principles do not simply relate claims about logical necessity and possibility (or validity) to claims about what one should or should not believe, but in fact relate *formal* claims about validity[27] to informal claims about deontic statements about beliefs. Bridge-principles are, therefore, not just connections between alethic

[26] The metaphysical status of the entities that are modelled by non-classical states is, for the sake of focus, deliberately left open.

[27] Here, I assume that the term *entailment* in Streumers proposal refers to a formally precise notion.

modalities and some combination of deontic and doxastic modalities, but also between formal and natural-language modalities.

By formalising the doxastic and epistemic modalities as claims about a particular logical space, we reduce the gap between the relata, but only as the result of a modelling decision, namely the adoption of a relatively high level of abstraction. This lesson is already implicit in the remark from Titelbaum (2008, 2013) that in the discussion of bridge-principles there are three rather than just two relata; I just accord more importance to the role of abstraction. It is crucial to factor in the abstraction step, because otherwise we may confuse the defeasible features of how information flows between different levels of abstraction[28] with the defeasible features of how logic (understood as a strict modality) is related to belief (a variable modality). Hence, the methodological recommendation that we shouldn't confuse the defeasible nature of the deontic modalities, with the effect of certain modelling choices.

5 PUTTING THE MODEL TO WORK

Logical information is, in the type of model we use, a kind of hard information: It is the hard information we have if we cannot distinguish any of the states in S from the actual state (equivalently: all the theorems of our logic), and this feature underlies the need for logical revision as well as the possibility of revisionary slides.

Let me illustrate this by making the explicit connection with the schematic revisionary argument from §3. The basic idea of such arguments is that if one can intuitively be committed to a set of beliefs that are both P and not-Q, but that logic precludes the existence of such sets because every P-set is a Q-set, then either we should drop the requirement that beliefs should be logically and deductively cogent, or revise logic such as to make room for P-sets that are not Q. When related to the structure of logical space, the claim that we can have certain beliefs maps to a claim about the existence of certain non-empty subsets of S. Namely that if we can have beliefs that are both P and not-Q, then the corresponding soft information should be non-trivial (i.e. be true in a non-empty subset of the logical space). As a consequence, if no such subset of S exists, we need to expand S to make room for such beliefs.[29] Call this the *dilution* of the logical space. Yet, diluting

[28] See Allo (2009) on the defeasible nature of reasoning between different levels of abstraction, with an application to the difference between the information we store and the propositional information we have.

[29] This is crucial: failures of deductive cogency are due to the absence of non-empty subsets of S wherein a set of formulae is true. This is a feature that is independent of the extension of how we represent an agent's hard and soft information within that space. It is about the possibility of modelling certain belief-sets as non-empty semantic propositions of hard and soft information. As a consequence, the outcome of a revisionary argument has to be modelled by a dilution of the logical space, and could only be modelled as a standard form of belief-revision by giving up the assumption that S is a model of logical possibility.

the space of possibilities means that either (i) we need to reject our initial assumption that S is a logical space (which is to reject that it is a model for logical possibility, and hence entails the rejection that logic constrains our beliefs), or (ii) we need to revise our logic.[30]

This type of reasoning is a common reaction to the problems of logical and deductive omniscience in the logics of knowledge and belief.[31] When it is a reaction to the possibility of rational failures of deductive cogency rather than merely a matter of bounded rationality, it is also susceptible to the possibility of revisionary slides. In this case, the outcome of such slides is not an almost empty logic, but an almost unconstrained space of logical possibilities; a situation where there is hardly a distinction between the logically possible worlds and the open worlds (worlds where there is no logical connection between typographically distinct formulae, see Priest (2005, §1.7)). In a model for explicit beliefs, open worlds are a valuable tool, but in a model of how logic imposes normative constraints on beliefs, the presence of open worlds simply defeats the idea of logical norms.

We can now take a look at how the position of the different characters from §1 can be recaptured in this setting. Quite naturally (see fn. 29), the simple revisionist will opt for the dilution of the logical space, and will thereby maintain that this space is still a model of logical possibility and necessity as well as a model of an idealised notion of knowledge and belief. As before, the success of the basic revisionist will entirely depend on the avoidance of a revisionary slide. Equally naturally, the basic sceptic will extend his scepticism to the possibility of a logic of knowledge and belief (as, for instance, in Hocutt 1972). No surprises so far. What about the critical sceptic and the sophisticated revisionist? Recall that the sophisticated revisionist argues for a structural revision of logic, a view of logic that leaves room for exceptions and the withdrawal of prior conclusions.[32] This, as we will see, resurfaces as a rejection of the view that logic is always a kind of hard information. Conversely, the critical sceptic acknowledges the gap between logical and rational norms for belief, but also endorses the possibility of realistic formal models of belief (and belief-change). This view is consistent with a dilution of the logical space, but with the clear proviso that the resulting modal space is no longer a model of logical possibility and necessity.

Here's a preview of the thesis I'll argue for: The critical sceptic and the sophisticated revisionist can agree that soft information lies at the basis

[30] This argument is only meant as a reformulation of the revisionary arguments we previously considered. Within the context of logics of knowledge and belief, the use of *fragmented models*, which gives up the requirement that an agent's information should correspond to a single set of indistinguishable states, is a third option (Stalnaker 1984, Allo 2013*b*).

[31] And some solutions to this problem even combine logical revision with the possibility of truly counter-logical states in S (e.g. Priest 2005, Chapt. 1).

[32] Velázquez-Quesada (2011, Chapt. 6) already models defeasible inference-forms as a kind of soft information, but doesn't treat this as a kind of logical information.

of our reasoning processes, but whereas the critical sceptic sticks with the orthodox view that no soft information is logical information, the sophisticated revisionist will uphold that not all soft information is extra-logical. Put in more familiar terms: While some or even most of logic is knowledge-like, some of it is merely belief-like.

♦ ♦ ♦

As in previous sections, a closer look at the dialectic between our protagonists further clarifies what's at stake (and how the border between simple revisionism and sophisticated revisionism and critical scepticism isn't absolute). Let us, at least for the remainder of this section, put the question of revisionary slides aside and take a closer look at the views of two proponents of logical revisionism, namely Graham Priest and Jc Beall.[33] As they see it, when it comes to deductive logic, **LP** (see §3) is the right choice: It doesn't just block the road between mere inconsistency and triviality, but it also prevents one to reason from truths to falsehoods. They also agree that, when it comes to actual reasoning, **LP** is simply too weak. The joint rejection of the *disjunctive syllogism* (DS) and *modus ponens* (MP)[34] just blocks too many common reasoning patterns. Their disagreement only arises when we look further at how the gap between logic and reasoning should be closed. (here, the gap is due to the fact that a given paraconsistent logic seems to *under-generate*).[35]

For Beall, we shouldn't close this gap by logical means, for he agrees with Harman that there is a conceptual difference between logical principles and norms for rational belief (Beall 2013*a*, 2014), and believes that since logic should never lead one from truth to falsehood (Beall 2012), no extension of **LP** that reintroduces MP or DS in one form or another is in fact acceptable.[36] In addition, he argues that a suitable *multiple-consequence* presentation of **LP** has all the resources to explain our actual uses of MP and DS as extra-logical inference-steps.[37] What he refers to is that, while MP and DS are invalid in **LP**, the following multiple-consequent argument-schemas are perfectly fine (the comma in the consequent can be read as a

[33]Revisionary slides do not pose a problem for them because their logical revisionism is more closely related to the semantic paradoxes than to the mere possibility of inconsistent beliefs. As they also believe that their preferred paraconsistent logic imposes rational constraints on what we may and may not believe, their views are no less relevant to the present discussion.

[34]Given the definition of the material conditional $A \supset B$ as $\neg A \vee B$, DS and MP are really just one inference-form.

[35]A similar story could be told in terms of the paraconsistent logic **CLuNs** and the disjunctive syllogism only. This is the familiar motivation for the development of adaptive logics. I only stick to the **LP**-version for expository reasons.

[36]This is in a sense not a pure claim about logic, but a claim about which logic is appropriate for reasoning from or constraining one's beliefs.

[37]I do not further go into his reasons for not adding a detachable implication, because they are related to Curry's paradox, and not to our present concerns. For present purposes it suffices to point out that Beall primarily wants to show why we can do without a detachable conditional, but that his proposed solution is also more generally applicable.

disjunction):

$$\neg A, A \vee B \vdash_{\mathbf{LP}} B, A \wedge \neg A \qquad \text{(DS*)}$$

$$A, A \supset B \vdash_{\mathbf{LP}} B, A \wedge \neg A \qquad \text{(MP*)}$$

and impose specific constraints on our beliefs. Namely that (for the latter case) we cannot simultaneously accept A, $A \supset B$, and reject both B and $A \wedge \neg A$. Since, moreover in **LP** we have:

$$\neg A, A \vee B \nvdash_{\mathbf{LP}} B \text{ and } \neg A, A \vee B \nvdash_{\mathbf{LP}} A \wedge \neg A$$

$$A, A \supset B \nvdash_{\mathbf{LP}} B \text{ and } A, A \supset B \nvdash_{\mathbf{LP}} A \wedge \neg A$$

DS* and MP* are, in Beall's parlance, *strict choice validities*, which, as constraints on our beliefs only present us with our logical choices or options. When in our reasoning we use MP or DS, we simply decide to dismiss the possibility that a contradiction might be involved, but this choice is itself not sanctioned by logic.

> Indeed, logic is silent on which of such given logically available options are rational options. Logic constrains the space of rationally available options—nothing more.
>
> (Beall 2013*a*, 5)

This view can be seen as a corollary of the logical orthodoxy that logic deals with possibility and necessity, and makes no further distinctions. That is, in the terminology I introduced, logic is just a kind of hard information. Interestingly, while Beall is a logical revisionist in his adoption of **LP**, his further views on how logic and reasoning are related bring him closer to the critical sceptic: Logic presents us with the options, but further extra-logical machinery (bridge-principles) is required if we want to model actual reasoning.

Presumably, Beall's views about *strict choice validities* imply a partial bridge-principle, because they relate logical options (i.e. what is logically possible) to doxastic options (what we may rationally believe, or what is doxastically possible. (And these are best understood as bridge-principles between formal modalities.) The main feature of this proposal is that the gap between logic and reasoning is acknowledged (indeed, with an explicit reference to Harman),[38] and that only non-logical principles can be invoked to bridge this gap. Furthermore, since doxastic options are insufficient to explain our actual reasoning, we also need an account of how we choose between these options.

In a different series of publications, Beall (2013*b*,*c*) develops such an account based on what he calls *shrieking*. With a "stronger-than-logic closure operator" that includes a set of extra-logical rules that allow one to derive an absurdity from a particular contradiction, the intended consistent

[38]Though it should be emphasised that Beall doesn't blindly follow Harman, as his views on the constraint-role of logic is more akin to that of Streumer (2007) on reasons for belief.

behaviour of certain formulae can be explicitly affirmed without including additional vocabulary that expresses this consistency.

For Priest, by contrast, adaptive logic shows us how we should bridge the gap between the relatively weak logic **LP**, and our actual use of many classically valid arguments that are invalid in **LP**, and the non-monotonic extension **MiLP** (for *minimally inconsistent*) of **LP** provides the precise implementation of this idea (Priest 2006*b*, Chapt. 16).[39] Given the **LP**-validity of (MP*), and the absence of evidence for the inconsistency of A, we may defeasibly derive B from A and $A \supset B$ *until and unless* one receives additional evidence (either in the form of new information, or through deduction) that A is inconsistent after all. This approach can be understood in many ways. We can think of it as the permission for defeasibly adding a background-assumption that expresses the consistency of A,[40] or as the permission to defeasibly apply classically valid rules (i.e. with the addition of a negative condition, as in the dynamic proof-theory of adaptive logics). If we stick closer to Beall's terminology of *strict choice validities*, we can equally well think of these as default preferences between our logical options. That is, in the present case, as the suggestion that the consistent option is our default *logical* option.

The proposed connection between Beall's strict choice validities, and the default-preference for the consistent option in adaptive logics like Priest's **LP** isn't far fetched at all: the standard monotonic characterisation of adaptive logics relies, as Theorem 1 below illustrates for **MiLP**, on the derivability of disjunctions in the weaker monotonic logic.[41]

THEOREM 1 (MONOTONIC CHARACTERISATION OF **MILP**) Where Ω is the set of all atomic inconsistencies, $\Sigma(\Gamma) = \{\Delta_1, \ldots, \Delta_n\} \subseteq \mathcal{P}(\Omega)$ such that for each Δ_i we have $\Gamma \vDash_{LP} \bigvee \Delta_i$ and $\Gamma \nvDash_{\mathbf{LP}} \bigvee \Delta_i'$ for any $\Delta_i' \subset \Delta_i$, and $\Phi(\Gamma)$ the set of minimal choice-sets of $\Sigma(\Gamma)$, we have:

$$\Gamma \vDash_{\mathbf{MiLP}} A \text{ iff for each } \phi \in \Phi(\Gamma) \text{ there is a } \Delta \subseteq \Omega \setminus \phi \text{ such that } \Gamma \vDash_{\mathbf{LP}} A \vee \bigvee \Delta$$

And since each $\Gamma \vDash_{\mathbf{LP}} A \vee \bigvee \Delta$ is clearly a strict choice validity, all that **MiLP** does is showing the *logical* way of making choices. When compared to Beall's *shrieking* proposal, a different general theorem about adaptive consequence-relations[42] reveals that any **MiLP**-closed theory will also be

[39] This does not mean that Priest believes that **MiLP** is a better logic than **LP**, but only that there is a logical account of our usage of the disjunctive syllogism and material detachment. Priest seems to present **MiLP** as a formal theory of reasoning (Priest 2006*b*, 16.2), or as an account of quasi-validity (8.6), but also suggests that it is universally applicable (Priest 2012), which is a feature we often associate with logical consequence.

[40] Something along the lines of Makinson (2003, §2). See also the discussion in Strasser & Antonelli (2014, §3.6).

[41] For the general version, see Batens (2007).

[42] The *Derivability Adjustment Theorem* which ensures that by considering as many inconsistencies false as Γ admits, the closure of Γ under an inconsistency-adaptive logic will be as close as possible to its classical closure without lapsing into triviality. For the general formulation of this theorem, see Batens (2007, 230).

closed under all unproblematic shrieking-rules. Indeed, in several publications Batens (2009, 2013) affirmed that adaptive logics can be seen as the inference-engine behind the addition of consistency statements in, for instance, da Costa **C** systems. These claims apply even more directly to Beall's proposal.

♦ ♦ ♦

The above digression shows that the disagreement between, on the one hand, the simple revisionist and the basic sceptic, and, on the other hand, the sophisticated revisionist, is really about the scope of logic: Does logic merely provide us with options (hard possibilities), or does it also choose certain options (defeasible preferences)? Getting back to our metaphor of logical space, is logic really just a kind of hard information, or is there also a sensible kind of soft logical information?

Let us, for the sake of answering this question, make precise what our logical space should look like when reasoning is based on **MiLP**. First, the total logical space S should be a model for **LP**-logical possibility: $s \in S$ *iff* the set of formulae that are true at s is **LP**-non-trivial. Consequently, an agent's hard information (what is true in all $s \in [s_0]_a^{\sim}$) is deductively closed under **LP**. Second, if this logical space is also a model for the logical preferences imposed by **MiLP**, there is a pre-order $\leq_{Ab}$ over S for which

$$s \leq_{Ab} t \text{ iff } Ab(s) \subseteq Ab(t) \tag{Ab}$$

with $Ab(s)$ and $Ab(t)$ the sets of atomic inconsistencies true at, respectively, s and t. Consequently, since $s \leq_{Ab} t$ holds *iff* s is not more inconsistent than t,[43] the $\leq_{Ab}$-minimal states in each subset of S will be the least inconsistent states in that set. If we then choose to identify an agent's soft information with the $\leq_{Ab}$-minimal states of $[s_0]_a^{\sim}$, this soft information will (as a corollary of a result from Allo (2013*a*)) be the **MiLP**-closure of his hard information. (Although similar results hold for a wide variety of adaptive logics, I shall stick to the **MiLP**-example for most of my exposition) The soft information encoded by (Ab) is only one type of soft information, and should be seen as a complement of an agent's soft factual information (beliefs about what is the case, information obtained from uncertain sources, *etc.*).

On the traditional picture I sketched in the previous section (§4), only two combinations are taken into account: (knowledge-closure) hard information closed under hard logical information, and (belief-closure) soft information closed under hard logical information. In both cases, this is just a vacuous intersection-operation of logical information with an agent's information: we take the intersection of, respectively, $[s_0]_a^{\sim}$ and the $\leq_a$-minimal states in $[s_0]_a^{\sim}$ with the total logical space S. That is, we just

[43] Where the 'more' is understood qualitatively (in terms of set-inclusion) instead of quantitatively (merely counting inconsistencies).

consider the hard and soft logical information of an agent a, and need not further worry about logic, because S simply doesn't include counter-logical options.

Once we add soft logical information, additional combinations arise, namely hard information closed under soft logical information (see above), and soft information closed under soft logical information. In the latter (soft/soft) case, there's more than one way to combine them, depending on the relative weight accorded to $\leq_a$ and $\leq_{Ab}$.

A first option is a limiting case, and only takes into account the $\leq_{Ab}$-states in S

NORMALITY a's soft information is the (possibly empty) intersection of the $\leq_a$-minimal states in $[s_0]^{\sim}_a$ with the $\leq_{Ab}$-minimal states in S.

Since $\leq_{Ab}$-states in S are just the classical states,[44] this is equivalent to closing a's soft information under classical logic. This approach is only of theoretical interest, since it easily trivialises inconsistent soft information.

The more interesting options do not rely on the intersection of sets of states, or on the straightforward selection of minimal states, but combine the underlying orderings. Lexicographic orderings are the standard approach in this case (I only present the basic case with two orderings):

DEFINITION 1 (LEXICOGRAPHIC ORDERING) The lexicographic ordering $\leq_{x,y}$ based on $\leq_x$ and $\leq_y$ is defined as:

$$<_{x,y} := <_x \cup (\approx_x \cap <_y),$$

with $<_x = \leq_x \cap \not\geq_x$, and $\approx_x = \leq_x \cap \geq_x$.

where the first ordering $\leq_x$ in $\leq_{x,y}$ takes precedence over the second ordering $\leq_y$.

By analogy with the closure of hard information under soft logical information, we can give precedence to an agent's soft information, and only use logical preferences (soft logical information) to resolve the remaining cases where $\leq_a$ is undecided.

BELIEF FIRST a's soft information is the set of $\leq_{a,Ab}$-minimal states in $[s_0]^{\sim}_a$.

This approach is, given a minor assumption about $\leq_a$, equivalent to taking a's soft information as premisses, and then closing it under **MiLP**:

If $\leq_a$ is connected in $[s_0]^{\sim}_a$, i.e. if there are no $\leq_a$-incomparable pairs of states in $[s_0]^{\sim}_a$, then

[44] Again, on the assumption that a sufficiently strong plenitude-principle holds. For instance that every non-trivial set of formulae is true somewhere in S.

1. $\approx_a$ is an equivalence-relation over $[s_0]^{\leq}_{\tilde{a}}$, and
2. s is $\leq_{a,Ab}$-minimal in $[s_0]^{\sim}_{\tilde{a}}$ iff s is $\leq_{Ab}$-minimal in $[s_0]^{\leq}_{\tilde{a}}$.

Although standard because it treats uncertain information as premises (and has 'normality' as a limiting case), this approach does go against the orthodox view that extra-logical information cannot trump logical information. A 'logic first' approach restores the traditional priority for logic:[45]

LOGIC FIRST a's soft information is the set of $\leq_{Ab,a}$-minimal states in $[s_0]^{\sim}_{\tilde{a}}$.

The 'belief-first' option is a rudimentary model of situations where extra-logical information trumps logical information. Indeed, if we think of $\leq_a$ as the aggregate of all of a's extra-logical information, the resulting model is one where uncertain information (be it of an empirical nature like evidence from an experiment or evidence obtained through testimony, or of a theoretical nature like views about what makes a good theory) is not discarded because it fails to meet the logical standard of consistency.

In section 2 I already dismissed the possibility that logic could have a special role in reasoning because logical norms always take precedence over extra-logical norms, and rational failures of deductive cogency precisely draw our attention to the defeasible nature of some of our logical norms. This idea is made precise by having soft logical information that cannot only be trumped by hard information, but also by soft information: the $\leq_{Ab}$-minimal states of the whole logical space need not be the best states of an agent's total information. Crucially, soft logical information is defeasible in view of how it is merged with our other information, and in that sense it doesn't conform with our traditional picture of logic.

However, it is not because we allow for soft logical information that we have to give up the distinction between logical and extra-logical soft information. Unlike mere beliefs, soft logical information (encoded as a pre-order on the logical space) has two crucial features: (i) it is a property of the logical space, and (ii) it is a purely formal feature.

To say that $\leq_{Ab}$ is a formal feature of a logical space is to say that it is both defined for the whole logical space (and thereby shared by all agents), and characterised in purely syntactical terms. More exactly, $\leq_{Ab}$ is a so-called formula preferential ordering (Ab) in the sense that being lower in the ordering means satisfying fewer formulae of a certain set, where this set is characterised by a logical form. In the case of **MiLP**, the set of relevant contradictions isn't just any set of inconsistencies, but the set of all inconsistencies that satisfy a certain schema (Viz. $A \wedge \neg A$ with A atomic). Furthermore, and perhaps more surprisingly, such properties of the logical space are unrevisable in the sense that they cannot be changed

[45] Logic first has a variant of 'normality' as a limit-case, namely one obtained by first taking the (possibly empty) intersection of $[s_0]^{\sim}_{\tilde{a}}$ with the $\leq_{Ab}$-minimal states in S, and then selecting the $\leq_a$-minimal states.

(while remaining purely formal orderings) without first requiring a dilution or reduction of the logical space itself.[46] This is an important insight (see the next section for a precise account), because it ties the specialness of logic not just to formal features, but also to its higher-order properties in reasoning-processes.

By drawing attention to the fact that logical orderings do not obey standard revision-principles, I am arguing that making logic more belief-like by denying that all logical information is hard information does not entirely obliterate the distinction between regular belief and soft logical information, or even between logical and extra-logical requirements on theories. Giving up the idea that all logical modalities are strict, means that logical information is merged with—rather than radically imposed on—other information. This severs the traditional link between logical and necessary consequence, but it doesn't make logical information any less a formal property of a logical space (and thus still a body of necessary formal truths). This, I submit, is sufficient to argue that logic imposes a special, or at least a different kind, of requirement on beliefs.

6 Revisionary slides and arguments revisited

By tying the discussion of logical options and logical preferences in the previous section to the views of Beall and Priest, the problem of revisionary slides was temporarily ignored. As logical revisionists, they both assume that revisionary slides can be stopped, and hence Priest's defence of **MiLP** does not have to appeal to specific properties of abnormality-orderings to defend the normative role of logic for reasoning. Once we take the possibility of revisionary slides seriously this changes, for the outcome of a revisionary slide is the absence of any hard logical information. Given the dialectic between our four characters, we cannot dismiss this possibility.

Revisionary slides that lead to the total dilution of the logical space do not pose a problem if the successive reductions of hard logical information are matched with increases of soft logical information; that is, if hard requirements are replaced by soft preferences. In the case of a revision of classical logic, this means that a classical space isn't merely replaced by an **LP**-space, but that the latter is also endowed with an appropriate abnormality-ordering. This strategy is only successful if the notions of soft logical information or logical preferences can be used to defend the special status of logic in reasoning. Once we give up the idea that logic is special because it provides the most general kind of hard information (linked to the total space of possibilities), we can only defend the special status of logic by showing that soft logical information differs in some crucial ways from soft extra-logical information.

[46] There are exceptions, like the total inversion of $\leq_{Ab}$, but these are barely relevant.

In the previous sections two static arguments were given: (i) logical preferences or abnormality-orderings are global, (ii) formal properties of the logical space, and one dynamic argument was briefly mentioned: (iii) logical revision does not fit the standard patterns of belief-revision because it isn't just a change in preferences, but always includes a dilution (or restriction) of the logical space.

Let me illustrate what this means by reconsidering the second revisionary argument from §3. Given a logical space based on **MiLP**, the upshot of this step is to avoid the further spreading of inconsistencies by invalidating the *De Morgan* equivalences that allow one to drive negation inwards.[47] In terms of the dilution of the logical space, this means that we will add possibilities where, for instance, $\neg(p \wedge q)$ is true but where $\neg p \vee \neg q$ is false. In the resulting space, an ordering $\leq_{Ab}$ that is only sensitive to differences in atomic contradictions will no longer be able to track all relevant degrees of inconsistency: s may intuitively be more inconsistent than t because it makes some complex contradiction $(p \vee q) \wedge \neg(p \vee q)$ true, and this may go unnoticed because $p \wedge \neg p$ and $q \wedge \neg q$ may both be false in s. This is easily fixed by lifting the restriction on $\leq_{Ab}$ and make it sensitive to all explicit contradictions.

The above example makes it clear that dilutions of the logical space should in general be accompanied by refinements of the logical preferences: if we have more logical options we also need more fine-grained logical preferences. But the converse principle is also true: finer logical preferences only make sense if more logical options are added as well. If, as in **LP**, every difference in arbitrary explicit contradictions is also a difference in atomic explicit contradictions, an ordering that is only sensitive to the latter type of contradictions will be identical to an ordering that is sensitive to the former type of contradictions. An even more blatant example is this: in a classical space of possibilities an ordering that tracks degrees of inconsistency will have no effect at all.

As show by the above examples, mere changes in logical preferences cannot arise, and logical revision can therefore not be modelled in terms of traditional revision strategies like conservative or radical upgrades that only affect the ordering of possibilities. This marks a clear difference between logical and extra-logical types of soft information.[48]

7 Concluding remarks

Though this paper engages with Harman's objections against the traditional view that logic provides norms for reasoning, it does so by simultaneously

[47] This yields a weakening of **LP** that is equivalent to the result of replacing the detachable implication in **CLuN** with an implication defined as $\neg A \vee B$.

[48] This argument relies on the assumption that there is only one logical space. In formal models where multiple logical spaces are part of a super-logical space (see Mares 2014, §12) the situation is different.

situating it within a broader argumentative context, and focusing on the problem of failures of deductive cogency. The development of a formal model that is based on the metaphor of logical space is in line with this focus, and has the benefit that the problem of clutter-avoidance can be put aside.

As a result, the proposal developed in this paper falls short of a full-fledged answer to Harman's challenge, but it also has a number of noteworthy theoretical virtues. These include the integration of the problems raised by rational failures of deductive cogency with those raised by the problem of logical revision, and the insight that logic can be specially relevant for reasoning even when it does not impose infallible norms.

An unexpected but central insight in this paper is the distinction between bridges between formal theories and the focus phenomenon they model, and bridges between logical and doxastic modalities. One of the issues with trying to meet Harman's challenge is related to attempts to provide a bridge-principle that does both at once. Once we recognise this gap (which is also acknowledged in the work of Titelbaum), it becomes clear why partial bridge-principles are more effective, and certainly more explanatory. The lexicographic combinations of factual and logical soft information (labelled 'belief first' and 'logic first') are best seen as partial bridge principles that relate formal logical modalities to formal doxastic modalities. Another bridge principle is implicit in Theorem 1, and relates strict logical modalities (logical options or hard information) to variable logical modalities (logical preferences or soft information). These are the bridge principles that do the major lifting with respect to rational failures of deductive cogency.

One way to look at my proposal is as a philosophical account of what non-monotonic logic is about, namely a formal account of soft logical information, and how this relates to reasoning and belief. On that account, we have both a non-trivial bridge between hard and soft logical information and a fairly direct connection between soft logical information and belief. Another way to look at it, is as a formal account of what the sophisticated revisionist and the critical sceptic can agree on, namely a highly idealised model of belief wherein logic—understood as the strict modality of entailment—plays a role, but only indirectly so via the notion of soft logical information. This can be applied to the disagreement of Beall and Priest over the status of **MiLP**, but also to the views of Harman, MacFarlane and Titelbaum.

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