Synonymy and Intra-Theoretical Pluralism

Comment on Hjortland (and a bit more)*

Abstract

The starting point of this paper is a version of intra-theoretical (logical) pluralism that was recently proposed by Hjortland (*AJP*, **91(2)**: 355–73.). In a first move, I use synonymy-relations to formulate an intuitively compelling objection against Hjortlands claim that if one uses a single calculus to characterise the consequence relations of the paraconsistent logic **LP** and the paracomplete logic **K3** one immediately obtains multiple consequence relations for a single language and hence a reply to the Quinean charge of meaning variance. In a second move, I explain how a natural generalisation of the notion of synonymy (adapted to the 3-sided sequent-calculus used by Hjortland) can be used to counter this objection, but also show how the solution can be turned into an equally devastating "one logic after all" type of objection. Finally, I propose the general diagnosis that these problems could only arise in the presence of conceptual distinctions that are too coarse to accommodate coherent pluralist theses. The latter leads to the general methodological recommendation that the conceptual resources used to think and talk about logic should be kept in line with the formal resources that are used to define and describe a logical theory.

Keywords: Logical Pluralism · Meaning Variance · Logical Discrimination · Synonymy · Multi-sided Sequents

*Although it is presented as a comment on a recent paper by Hjortland, this contribution to the pluralism debate is really meant to advance my own investigation of the nature and role of logical discrimination and synonymy in our thinking about logic. Hjortland's formulation of intra-theoretical pluralism not only forced me to revise some of my own views, but n-sided sequents also happen to be nice case-study.

1

Logical pluralism isn't a unified position. There are many ways of spelling out in detail what one could mean by the claim that there is more than one true logic. As emphasised in Field (2009), the latter doesn't mean that all variants are equally valuable; only pluralisms that are both true and interesting (and perhaps even surprising) are really worth pursuing. By spelling out the desiderata for logical pluralisms, we do not only explain what we mean by interesting and true, but we also provide criteria for telling different forms of logical pluralism apart.[1]

For the purpose of this paper, I want to consider the following three criteria in particular: (i) logical pluralism is only interesting if it is *genuine*, that is, if the plurality of logics does not arise from mere verbal disagreement; (ii) it is only "true" if it is *stable*, i.e. immune to a "one true logic after all" type of objection; and in addition to these (iii) it is only worth pursuing in general if it is *useful* in the sense of providing a good account of logical practice.

The reference to logical practice in the last criterion is rather broad. It refers simultaneously to deductive practice (reasoning we qualify as logical), and to the scientific study of this deductive practice (codification, formalisation, ..., or, more generally, "what logicians do"). The inclusion of the scientific practice is essential, since pluralism about logic is to a large extent a pluralism about theories of deductive practice. As such, to be a pluralist is to accept that there are equally good accounts of correct deductive practice, in the sense that there can be equally good descriptions of a single practice (or set of norms), but also that there can be equally good practices (or sets of norms).

The distinction between a deductive practice and a theory of this practice is more important than the distinction between descriptive and normative theories. We can disagree about whether a given theory gets the norms right, but we can also disagree about whether a theory is descriptively adequate.[2] Analogously, we can be pluralists about getting the norms right or about giving a correct description. While important for our thinking about logic in general—it does matter whether logical revision is a form of theory-revision or an actual reformation of the norms for correct deductive inference—, such distinctions are less important for the issues I want

[1] I do not at all aim at a complete classification of types of logical pluralism. Many of the pluralisms that are reviewed by Cook (2010) and Russell (2013) do not directly fit into the schema I propose.

[2] Though note the inherent ambiguity in the phrase "getting the norms right," as this can itself refer to making correct normative claims, and making correct descriptive claims about pre-existing norms.

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to raise in this paper. The crucial import of the third criterion is that, as claims in the philosophy of logic, pluralist theses are claims about actual everyday and scientific practices, and as such they should not misrepresent these practices.

Let us, for the sake of argument, assume that the holy grail of logical pluralism is a position that coherently combines these three features, and—given the influence of the Quinean *meaning-variance* thesis—especially avoids the trap of verbal disagreement (Quine 1986: Chapt. 6).[3] The type of pluralism defended in Beall and Restall (2006) is meant to fit this description, and so is the intra-theoretical pluralism that was recently defended by Hjortland (2013). The contrast between these two proposals can be further clarified by bringing in a third contender, namely the logical pluralism advocated by Carnap. According to this view, which results from his principle of tolerance (Carnap 1971: §17) together with the view that the meaning of the expressions in a language is entirely determined by the formation and transformation-rules for that language (Carnap 1971: e.g. §1), we are not only free to choose our language and logic, but any logical difference results from a difference in language (Restall 2002: 431). Carnap's pluralism thus fares particularly well on the second and third criterion, but fails at (or rather explicitly dismisses) the first criterion.

For Carnap, setting up a language and setting up a logic are just two sides of the same coin (Eklund 2012). A language, if properly defined, comes with its own logic, and from that perspective the idea of having different logics within the same language just doesn't make sense. For Beall and Restall (2006), by contrast, a logic, and by extension a logical theory, is just made up from a language and a consequence relation that is defined over that language. As a consequence, while on that account it surely makes sense to define multiple consequence relations over one and the same language, the identification of logics with consequence relations that Beall and Restall put forward doesn't obviously allow for multiple consequence relations *within* a single logic or logical theory.[4] The idea behind intra-theoretical pluralism is to let go the latter identification (one logical

[3]Paoli summarises this view as follows: When confronted with a deviant system formulate in a language with a syntax just like that of classical logic (i.e. with a negation-like unary connective, with conjunction, and disjunction-like binary connectives, etc.), there are two ways of evaluating apparent departures from classical logic. One can either trust the homophonic translation proposed by the deviant logician, or conjecture that the deviant logician really means something different, and that a heterophonic translation could be used to avoid the apparent conflict with the theses of classical logic. Quine suggests that the *principle of maximal agreement* should lead us to favour the latter option. (Paoli 2003: 538)

[4]I'm here assuming that Beall and Restall's identification of logics with consequence relations extends to the identification of logics with logical theories, and hence also to the identification of logical theories with consequence relations.

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theory = one consequence relation) as well. By looking at logical theories as formal calculi (ways of setting up formal deductions), the multiple use of a single calculus to characterise two or more consequence relations suffices to drive a wedge between logical theories and consequence relations.

The above way of carving out the different types logical pluralism rests on the following insight: Quite like non-classical logics are obtained by letting classically indistinguishable formulae come apart, so do different guises of logical pluralism arise by extending and generalising the conceptual toolbox we use to think and talk about logic and/or logics. This idea of refining our conceptual apparatus is a recurrent theme throughout this paper.[5]

There are at least two ways of enriching our conceptual toolbox: We can create new concepts from scratch, for instance by deploying existing techniques in new ways. Alternatively, we can further refine concepts that we already have by letting two or more of their traditionally equivalent guises come apart. In this paper, I shall start by using the first approach, and will end by drawing attention to the second approach. To be precise, I start with exploiting the notion of *synonymy* to expose some potential troubles for Hjortland's version of intra-theoretical pluralism, and end with a moral about how finer conceptual distinctions are needed to save logical pluralism from outright incoherence.

2

The first formulation of intra-theoretical pluralism is due to Restall (2008), who suggests that classical, intuitionist and dual-intuitionist consequence-relations can happily live together in a single sequent-calculus. This is because a single set of (object and structural) rules is common to all three consequence-relations, while additional constraints on sequents are the only differentiating factor (single-conclusion for intuitionistic logic, single-premiss for dual-intuitionistic logic, and unrestricted for classical logic).

In Hjortland's version, the possibility of having multiple consequence relations in a single theory can be deduced from the fact that:

[5]The idea that similar notions can be distinguished through careful analysis as a means to avoid equivocation or resolve inconsistencies is fairly common. Here, however, I'm not primarily referring to distinctions that we become aware of through careful consideration of our pre-theoretical insights (conceptual analysis), but rather to distinctions that become available by active intervention (conceptual engineering). That is, I'm alluding to the possibility to create finer distinctions by adding to our conceptual resources; not to the possibility to reveal existing distinctions with our actual conceptual resources.

¬	
1	0
i	i
0	1

∧	1	i	0
1	1	i	0
i	i	i	0
0	0	0	0

∨	1	i	0
1	1	1	1
i	1	i	i
0	1	i	0

FIGURE 1: 3-valued matrices

- Two important non-classical logics, namely the paraconsistent logic **LP** and the paracomplete logic **K3**, are based on the same 3-valued matrices (Figure 1),
- the technique of n-sided sequents (Baaz et al. 1993), which associates places in a sequent with truth-values, can be used to exploit the former fact to obtain uniform particle rules for the connectives of both these logics (Figure 2), and
- the divergence in the interpretation of the intermediate value (*gappy* and thus non-designated in **K3**, but *glutty* and therefore designated in **LP**) can be used to relate derivable 3-sided sequents to two distinct consequence-relations by stipulating that the derivability of $\Gamma \mid \Gamma \mid \Delta$ coincides with $\Gamma \vdash_{\mathbf{K3}} \Delta$ whereas the derivability of $\Gamma \mid \Delta \mid \Delta$ coincides with $\Gamma \vdash_{\mathbf{LP}} \Delta$.[6]

When combined with the inferentialist thesis that the meaning of the logical connectives is captured by their introduction and/or elimination-rules in an appropriate calculus, these three features suffice to conclude that both these logics agree on the meaning of the logical vocabulary, but disagree on the extension of *follows from*.

$$\frac{\Gamma_0, A \mid \Gamma_i \mid \Gamma_1 \quad \Gamma_0, B \mid \Gamma_i \mid \Gamma_1}{\Gamma_0, A \vee B \mid \Gamma_i \mid \Gamma_1}\vee_0 \qquad \frac{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, A, B}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, A \vee B}\vee_1 \qquad \frac{\Gamma_0 \mid \Gamma_i, A, B \mid \Gamma_1 \quad \Gamma_0, A \mid \Gamma_i, A \mid \Gamma_1 \quad \Gamma_0, B \mid \Gamma_i, B \mid \Gamma_1}{\Gamma_0 \mid \Gamma_i, A \vee B \mid \Gamma_1}\vee_i$$

$$\frac{\Gamma_0, A, B \mid \Gamma_i \mid \Gamma_1}{\Gamma_0, A \wedge B \mid \Gamma_i \mid \Gamma_1}\wedge_0 \qquad \frac{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, A \quad \Gamma_0 \mid \Gamma_i \mid \Gamma_1, B}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, A \wedge B}\wedge_1 \qquad \frac{\Gamma_0 \mid \Gamma_i, A, B \mid \Gamma_1 \quad \Gamma_0 \mid \Gamma_i, A \mid \Gamma_1, A \quad \Gamma_0 \mid \Gamma_i, B \mid \Gamma_1, B}{\Gamma_0 \mid \Gamma_i, A \wedge B \mid \Gamma_1}\wedge_i$$

$$\frac{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, A}{\Gamma_0, \neg A \mid \Gamma_i \mid \Gamma_1}\neg_0 \qquad \frac{\Gamma_0, A \mid \Gamma_i \mid \Gamma_1}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1, \neg A}\neg_1 \qquad \frac{\Gamma_0 \mid \Gamma_i, A \mid \Gamma_1}{\Gamma_0 \mid \Gamma_i, \neg A \mid \Gamma_1}\neg_i$$

FIGURE 2: 3-sided connective rules

The above conclusion bears some resemblance to a similar feature of substructural logics, namely the fact that many such logics only diverge from classical logic at the level of the structural rules, but are in agreement

[6] In fact, since $\vdash_{\mathbf{CL}} = \vdash_{\mathbf{K3}} \cup \vdash_{\mathbf{LP}}$, the same calculus can also be used to characterise the classical consequence relation.

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$$\frac{\Gamma_0, A \mid \Gamma_i \mid \Gamma_1 \quad \Gamma_0 \mid \Gamma_i \mid \Gamma_1, A}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1}\,(\mathrm{Cut}_{0,1}) \qquad \frac{\Gamma_0, A \mid \Gamma_i \mid \Gamma_1 \quad \Gamma_0 \mid \Gamma_i, A \mid \Gamma_1}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1}\,(\mathrm{Cut}_{0,i}) \qquad \frac{\Gamma_0 \mid \Gamma_i, A \mid \Gamma_1 \quad \Gamma_0 \mid \Gamma_i \mid \Gamma_1, A}{\Gamma_0 \mid \Gamma_i \mid \Gamma_1}\,(\mathrm{Cut}_{i,1})$$

FIGURE 3: 3-sided cut rules

with classical logic when it comes to the particle rules. Paoli (2003) relies on this feature to argue that, given a suitable calculus like the sequent-calculus **LK** introduced by Gentzen, we can answer the Quinean charge of meaning-variance.

However, the view that we can modify the extension of "follows from" by adding or dropping structural rules, *and* at the same time leave the meaning of the connectives intact, does presuppose that if two parties agree on the operational rules they *ipso facto* use the same connectives, and thus should rely on a homophonic translation to evaluate each other's claims. For this to be viable, one either needs to deny that structural rules contribute to the meaning of logical constants (as, for instance, on the *nihilistic view*, see Paoli (2002: 9)), or that meaning-*in*variance can be enforced by merely keeping the core or local meaning (i.e. what is determined by the operational rules only) of the connectives fixed. The main virtue of intra-theoretical pluralism is precisely that it doesn't have to settle the issue of how structural rules contribute to the meaning of our logical vocabulary. Because there's a single calculus, there is only one set of rules and only one set of correct proofs.[7]

3

At first blush, intra-theoretical pluralism scores high on all three criteria we proposed. My first attempt to cast doubt on these appearances depends on the following assumption:

M/S-ASSUMPTION Two logics agree on the meaning of the logical connectives *iff* they also agree on which expressions are synonymous.

The main reason for introducing synonymy into this debate is a Quine-like "no entity without identity" consideration; we can't give an account of meaning without also giving an account of sameness of meaning. The M/S-assumption is itself motivated as follows: If (i) synonymy is just sameness of meaning, and (ii) if sameness of meaning can only depend on logical

[7]Arguably, in Restall's original version of intra-theoretical pluralism we have different sets of correct proofs. A similar situation arises in an alternative version of n-sided sequents that relies on multiple accounts of axiomatic sequents (Degauquier 2012).

form and the meaning of the logical connectives, then (iii) one cannot agree on the meaning of the logical constants without also agreeing on the sameness of meaning of any two formulae A and B, and thus (iv) agreement on the meaning of logical constants is necessary and sufficient for agreement about which expressions are synonymous.

On a fairly standard—and because of the restriction to formal languages entirely unproblematic—explication of synonymy (Humberstone 2005, Smiley 1962), the expressions A and B are synonymous relative to $\vdash$ iff:

$$C_1(A), \dots, C_n(A) \vdash C_{n+1}(A) \text{ iff } C_1(B), \dots, C_n(B) \vdash C_{n+1}(B), \qquad (\mathrm{Syn}_{\vdash})$$

where each $C_i(B)$ is the result of replacing zero, one or more occurrences of A in $C_i(A)$ by B.

This is enough to cause trouble for intra-theoretical pluralism: Since all instances of excluded middle are synonymous relative to $\vdash_{\mathbf{LP}}$, but not synonymous relative to $\vdash_{\mathbf{K3}}$, and conversely all contradictions are synonymous relative to $\vdash_{\mathbf{K3}}$, but not synonymous relative to $\vdash_{\mathbf{LP}}$, it appears that **LP** and **K3** effectively disagree about the meaning of the logical connectives.

It isn't all that hard to see that this argument fails, for tying synonymy to consequence-relations really begs the question against intra-theoretical pluralism. To begin with, one can point out that, given some minor assumptions, a consequence-relation $\vdash$ is explosive *iff* any two contradictions are synonymous relative to $\vdash$.[8] Clearly, this makes it quite hard to maintain that what the paraconsistent logician means by "not" coincides with the classical logician's use of negation. More generally, since in **K3** and **LP** $\vdash$-synonymy and logical equivalence[9] are co-extensive, and changes in $\vdash$-synonymy are always mirrored by changes in what can and cannot be derived,[10] we simply cannot use ($\mathrm{Syn}_{\vdash}$) to track the kind of agreement between **LP** and **K3** that is reflected by their common proof-rules. Jointly, these two considerations indicate that ($\mathrm{Syn}_{\vdash}$) is just too coarse to be fruitfully combined with intra-theoretical pluralism.

[8]Starting from the identity $p \wedge \neg p \vdash p \wedge \neg p$, we only need to use the synonymy of $p \wedge \neg p$ and $q \wedge \neg q$ to first deduce $p \wedge \neg p \vdash q \wedge \neg q$, and then $p \wedge \neg p \vdash q$.
Conversely, given explosion we have $p \wedge \neg p \vdash q \wedge \neg q$ for any p and q, and this leads directly to synonymy through an application of the *cut*-rule.

[9]Since our language does not have an equivalence-relation, I reserve the term "equivalence" for the two-sided turnstyle $\dashv\vdash$ which, strictly speaking, is the relation of interderivability.

[10]See Humberstone (2005) on when the inverse relationship between logical discrimination (or synonymy) and deductive strength does and doesn't hold.

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4

Differences in logic can, and often should, be understood as differences in what can and cannot be told apart. Indeed, we often like to think of classical logic as fudging some distinctions that are available to, for instance, the intuitionist logician. These are the differences that ($\text{Syn}_{\vdash}$) registers, but these should not necessarily be the differences that co-vary with the meaning of the connectives.

Synonymy-relations can be used to track logical discrimination, but their original purpose lies elsewhere. Smiley (1962) introduced the formal notion of synonymy to answer questions related to the definability of expressions in a logical system. Naturally, if we think of a logical system as a consequence-relation $\vdash$ over a language $\mathcal{L}$, we can say that a certain expression $A \notin \mathcal{L}$ is definable in $(\mathcal{L}, \vdash)$ iff it is $\vdash$-synonymous with an expression $B \in \mathcal{L}$. In many cases this will coincide with there being a $B \in \mathcal{L}$ that is equivalent to A, but this need not always be the case. This, for instance, happens when synonymy and equivalence come apart (again, see Humberstone 2005), but it can also happen when we drop the familiar identification of logical theories with consequence-relations over a language. Indeed, while the respective $\vdash$-synonymy and equivalence relations coincide both in **LP** and in **K3**, neither of these relations will (on its own) tell us when an expression is definable in a 3-sided sequent-calculus. Studying definability within such a system calls for a further generalisation of synonymy. I submit that getting a better grip on the meaning of the connectives in such a system requires a similar generalisation of the synonymy-relation.

A natural generalisation of ($\text{Syn}_{\vdash}$) stipulates that A and B are 3-synonymous iff:

$$\begin{gathered}\Gamma_0, C_1(A) \mid \Gamma_i, C_2(A) \mid \Gamma_1, C_3(A) \text{ is derivable} \\ \text{iff} \\ \Gamma_0, C_1(B) \mid \Gamma_i, C_2(B) \mid \Gamma_1, C_3(B) \text{ is derivable}\end{gathered} \tag{3-Syn}$$

with, as before, each $C_i(B)$ the result of replacing zero, one or more occurrences of A in $C_i(A)$ by B.

This proposal builds on a distinction between 1-synonymy and 2-synonymy that is already present in Humberstone (2005). Whereas 2-synonymy ($\text{Syn}_{\vdash}$) is tied to the preservation of consequence, 1-synonymy only requires the preservation of theoremhood. Clearly, if one thinks of a logic as a set of theorems, the intended class of distinctions is captured by the coarser notion of 1-synonymy; if one thinks of a logic as a consequence-relation, the intended class of distinctions is captured by the more fine-grained notion of 2-synonymy. By analogy, if we have a theory that is formulated in terms of three-sided sequents, the corresponding notion of synonymy needs to be even more fine-grained.

What is crucial for our purposes is that the proposed relation of 3-synonymy unifies the two relations of 2-synonymy that are associated with the consequence-relations of **K3** and **LP**.[11] As Theorem 1 (see appendix) confirms, A and B are 3-synonymous iff they are equivalent in both **LP** and **K3**, and this is what, at least at first sight, allows us to sidestep the problematic conclusion of the previous section.

Let us first take a look at some specific details of the intersection of **K3** and **LP**. Since classical logic coincides with their union, it is very tempting to think that their intersection would coincide with a 4-valued system with two intermediate values (a designated one as in **LP** and a non-designated one as in **K3**); a logic commonly referred to as **FDE**.[12] This is not the case! If we have 4 values, we can provide a counter-example for the classically valid $p \wedge \neg p \vdash_{\mathbf{CL}} q \vee \neg q$, but no such counterexample can be found if we only have 3 values. If the intermediate value is designated, there is no way of making $q \vee \neg q$ non-designated; if the intermediate value is non-designated, there is no way of making $p \wedge \neg p$ designated. This result carries over to the intersection of the equivalence-relations of **LP** and **K3**. With 4 values it is easy to give $(p \vee \neg p) \vee (p \wedge \neg p)$ and $(p \vee \neg p) \vee (q \wedge \neg q)$ distinct truth-values, but this can't be achieved with only 3 values.[13]

Though it has some strange properties, 3-synonymy is a well-defined synonymy relation that can be used to salvage meaning-invariance, but that also poses a new threat to the stability of intra-theoretical pluralism: If A can only follow from Γ in virtue of what A and the premises in Γ mean, which, in virtue of the formality of logic, can be retraced to the meaning of the logical connectives that figure in A and in the premises in Γ, shouldn't we then conclude that having a single synonymy-relation is necessary and sufficient for having a single consequence relation as well? In the next section I consider the presuppositions of this question in more detail.

5

Traditional conceptions of provability and validity like the Tarskian characterisation of logical consequence as truth-preservation in all cases, or the

[11]In a sense, 2-synonymy also generalises two forms of 1-synonymy, namely one that preserves theorems and one that preserves counter-theorems (contradictions). I do not further pursue this analogy.

[12]For *first-degree-entailment*, or the first-degree fragment of many relevant systems.

[13]If the intermediate value is designated, we end up with two theorems; if the intermediate value is non-designated, both formulae are equivalent to $p \vee \neg p$.

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standard view that proof-steps should both be analytic and formal, are all too easily used to defeat logical pluralism. For instance, if we really need to take into account *all* cases, then surely any consequence relation that is based on a more restricted class of cases will fail as an account of logical consequence (this is a standard objection directed at Beall & Restall; see Priest (2001)). Similarly, any calculus that, in addition to a proof-theoretic characterisation of the meaning of the logical connectives, depends on further constraints and permissions on what can and cannot be derived will again fail as an account of what can and cannot be shown to hold in virtue of the meaning of our logical vocabulary. Going back to our example of substructural logic, if we use the operational rules of **LK** to characterise the meaning of the different connectives, how then can we motivate a consequence relation that is any stronger than that of linear logic?

As mentioned at the end of §2, intra-theoretical pluralism avoids an important part of this problem because it gives a single account of what can and cannot be derived (concretely: the structural rules of weakening and contraction can be taken to contribute to the meaning of the connectives because they are common to **LP** and **K3**). By focusing on the more abstract notion of synonymy, and developing 3-synonymy as the right kind of synonymy for a 3-sided calculus, it now seems that we have merely pushed the problem to a higher level. The generalised notion of 3-synonymy can now be used to argue that the meaning of the connectives only warrants a notion of consequence (and equivalence) that leaves the semantic status (designated or not) of the intermediate value indeterminate, and by Theorem 1 coincides with the intersection of **LP** and **K3**.

There is a certain recurring (and perhaps even all too familiar) pattern in how orthodox views about logic are used as objections against logical pluralism. I think we can (and should!) no longer ignore that these views have now become tools for forcing the collapse of some of the crucial distinctions that safeguard pluralist theses from sheer incoherence. Of course, such moves are often meant to show that the new distinctions are *ad hoc* and only meant to block the obvious conclusion that certain types of logical pluralism are plainly contradictory. This diagnosis might have some force from the perspective of conceptual analysis, but loses its force if we stick to conceptual engineering.[14] Let me show how this can be put to work.

When given a logical theory that is based on a 3-sided sequent calculus, we can reason about consequence and meaning at multiple levels. We can focus on which 3-sided sequents are derivable. This is the level at which

[14] The contrast between conceptual analysis and conceptual engineering is borrowed from Floridi (2011). I go deeper into the application of that distinction in the philosophy of logic in the final section.

the meaning of the connectives is fully characterised by the operational rules, and where our newly introduced relation of 3-synonymy adequately captures sameness of meaning. Alternatively, we can focus on what follows from what in a standard 2-sided setting. This is where the consequence-relations of **K3** and **LP** can be characterised, but also where ($\mathrm{Syn}_{\vdash}$) is the appropriate notion of synonymy.

The objection we have considered in the previous paragraphs confuses these two levels.[15] It tries to reinstate logical monism by freely moving between the 2-sided and the 3-sided perspective, and, more exactly, uses the unique 3-synonymy relation to obtain a single consequence-relation that has only 2 sides! As already suggested, this means that the semantic status of the intermediate value is itself left indeterminate in a formal setting where that value is presupposed to pick a side. This makes the resulting consequence-relation suspicious. By adopting the general framework of n-sided sequents, we do not only commit to the generalisation of the notion of synonymy (my main point from §4), but also to the idea that the number of sides coincides with the number of semantic values. Specifically, in a 3-sided setting we have 3 independent values (truth-values), and these can be used to characterise 2-sided consequence relations that are based on 2 independent values (the semantic values *designated* and *non-designated*).

Here too, we might, with a reference to the contrast between real and merely formal distinctions, ask if the above argument doesn't accord too much importance to the formal features of the n-sided framework where, as it happens, there is always a one-to-one correspondence between semantic-values and syntactic locations in a sequent.[16] Because we already implicitly allowed that the intermediate value could be semantically overdeterminate (designated and non-designated),[17] we might think that the dual decision of letting that value be under-determined is equally innocuous. Despite the initial analogy (underdetermination vs. overdetermination, intersection vs. union), the consequence-relation defined by $\vdash_{\mathbf{LP}} \cap \vdash_{\mathbf{K3}}$ remains rather unnatural. It is in the first place not at all clear why one would want to adopt it as an all-purpose logic since it is neither full-blooded partial, nor full-blooded paraconsistent. Considerations about the adoption of a logic aside (e.g. what kind of deductive standards may be codified by $\vdash_{\mathbf{LP}} \cap \vdash_{\mathbf{K3}}$), the problem remains, however, hard to pin down. I can only

[15]As remarked by a referee, this confusion makes sense if one thinks that sameness of meaning is just mutual meaning-containment, but this connection could just as well be abandoned.

[16]But see Restall (2009) for further results on the connection between two-sided sequents and having two truth-values.

[17]This is just another way of saying that the middle position is to be left empty, which makes the resulting consequence-relation classical.

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suggest that the ever-present indeterminacy (e.g. negation is either exclusive or exhaustive, but not both) and the need to resort to a reasoning by cases to characterise the extension of a consequence-relation reveal that with 3 truth-values, but only two sides we should not try to settle for a single consequence-relation.[18]

To sum up, with only 3 sides or 3 values available, the stability of intra-theoretical pluralism is not immediately under threat. Or it is at least immune to a certain type of charge *if* we do not make the further step from a single intermediate value with an indeterminate semantic value to two intermediate values, each with a determinate semantic value (as in the logic **FDE**). I briefly consider this option at the end of the following section.

6

Until now, I have primarily looked at how instability and meaning-variance threaten logical pluralism. In doing so, I repeatedly revealed that pluralist theses are hard to formulate, and that traditional conceptions of logical consequence often get in the way. In the final two sections, I'd like to put the third desideratum of *usefulness* at the forefront. As a philosophical position, logical pluralism only makes sense if it can be used to explain what logic is about and what logicians do.[19] I believe that we can make the required changes to our thinking about logical pluralism if we take this demand seriously.

According to the received view, we adopt a logic because of the entailments it validates and invalidates. On this view, logical pluralism is attractive because it vindicates different standards of deduction: Sometimes we want to extract as much information as we can from our premises, but on other occasions we need to be more careful. This is not the only story we can tell. According to an equally compelling view, we adopt a logic because of the distinctions it allows for. On that view, logical pluralism is attractive because it vindicates different ways of carving out contents: When, for instance, we reason about someone's beliefs we often want to keep beliefs that are classically equivalent apart. Of course, these two views are not exclusive. We often settle for a given logic because it provides the right

[18]In a sense, reasoning about $\vdash_{\mathbf{LP}} \cap \vdash_{\mathbf{K3}}$ requires one to reason about different precisifications (compare with the use of supervaluations in logics for vagueness) in a context where we don't expect to resort to such measures. See also my (2013) for a detailed discussion of reasoning about equivocal connectives.

[19]This focus on an actual practice is in the first place influenced by the philosophy of mathematical practice (Mancosu 2011). A generalisation to logical practice is less common, but equally revealing.

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mix of deductive strength and available distinctions (Allo and Mares 2012). And indeed, the different notions of synonymy and the widely valid inverse relationship between deductive strength and logical discrimination connect both concerns. This means that in some circumstances we give up certain distinctions to maximise the information yield of our premises, while in other circumstances we happily settle for a weaker logic just to be able to draw finer distinctions.

The resulting perspective on logic resembles Shapiro's logic-as-modelling view (Shapiro 2011, Cook 2010), but the focus on logical discrimination also allows for a very straightforward statement of the problem we repeatedly ran into. To be a logical pluralist is to give oneself a certain freedom to make some distinctions and/or to fudge some other distinctions, and this requires us to individuate contents more or less finely. But this also means that if we want to comply with the requirement of meaning-invariance, meaning and content should somehow come apart.[20] The objections we considered have one thing in common; they let the notion of synonymy double-task as "sameness of meaning" and as a criterion for logical discrimination. This is intuitively acceptable, for whenever synonymy and logical equivalence coincide, meaning and content appear to coincide as well. Yet, this feature is also the main obstacle for the formulation of a coherent pluralist thesis.

Once we recognise the need for such a distinction, we can see how intra-theoretical pluralism can live up to the expectations. The newly introduced notion of 3-synonymy does capture "sameness of meaning": it is only sensitive to the meaning of the connectives as they are defined by the particle rules, and because it is prior to the determination of the semantic status of the intermediate value, it is not suited as a criterion for logical discrimination. By contrast, the 2-synonymy relations defined by ($\mathrm{Syn}_{\vdash}$) do correctly track logical discrimination, but are too coarse as a criterion for sameness of meaning (at least when seen from the perspective of the calculus in which the logical theory is formulated).[21] This is all good news, but it also means that the different features we associate with logical pluralism are really associated with different parts of a logical theory (i.e. the calculus vs. the consequence relations). More radically, it also means that our thinking about traditional logical concepts like validity, derivability and meaning should be refined such as to match the formal resources of the logical theory we rely on.

[20]This was already one of the conclusions of my (2007), but here it is almost immediate given the previous assumptions I made about logical discrimination and synonymy.

[21]This shouldn't even be controversial, for it is a standard view that logical equivalence is just too coarse as a criterion for sameness in meaning.

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The view that our conceptual resources should follow our formal resources has several ramifications. For instance, it entails that the claim that logic is analytic is no longer unambiguous, for it can refer to the view that logical consequence is truth preservation in virtue of the meaning of the logical connectives, but also to the view that the content of the conclusion of a valid argument should not exceed the combined content of its premises. As we have seen, these two characterisations may come apart, and this lets us reconcile meaning-invariance with a stable form of logical pluralism (one language, many consequence relations).

One way to think about this split relates the meaning of the connectives to the *in-principle available* distinctions of a formal language, and the way in which contents are individuated (and the extension of meaning-containment relation) to the *actual distinctions* that are retained once we factor in the effect of logical equivalence. In some cases, the available distinctions are not sufficient to characterise a sensible (2-sided) consequence relation, and further choices must be made. This is the situation we described in the previous sections. In other cases, the available distinctions are sufficient to pick out a consequence relation. This is, for instance, the case with a 4-sided calculus where the rules for the connectives unambiguously pick out the consequence relation of **FDE**. Crucially, since keeping all of the available distinctions is already a decision about the individuation of contents, even with 4-values/sides it can make sense to collapse some of the available distinctions.

7

Whereas the avoidance of meaning-variance is explicitly at odds with Carnap's pluralism, there is nevertheless a distinctive Carnapian flavour in my defence of logical pluralism. This especially comes to the surface in the claim that we should let our conceptual tools follow the development of our formal tools. When I advance conceptual engineering as an alternative (or complement) to conceptual analysis in the philosophy of logic, I go with Carnap by preferring a *logic-first* to a *philosophy-first* approach in our thinking about logic. By this I mean that, where Carnap suggests that we shouldn't build a logic from first principles (Russell 2013), the evaluation of our formal systems shouldn't exclusively be based on first principles (understood as pre-theoretical conceptions) either. This doesn't mean that logical investigations cannot be guided by philosophical motives, or that pre-theoretical insights should be put aside, but rather that when we develop a logic, we shouldn't just rely on our pre-theoretical insights to evaluate this logic, but also acknowledge that the logic we developed is meant to refine our thinking about key logical concepts.

This type of approach may seem suspect, for what I suggest is that certain distinctions, like the distinction between meaning and content, do not need to be motivated solely in terms of our pre-theoretical understanding of these terms,[22] but can originate from a distinction that becomes available through the adoption of a certain formalism. All too often we think of these distinctions as purely formal features that cannot play a philosophical role (cfr. the distinction between *pure* and *applied* semantics), but there's really no reason why this should be so. After all, we do not only accept that the distinctions afforded by a formal theory do not always coincide with the distinctions that we tend to accept (i.e. formal theories can not only be abstractions and/or idealisations, but can also function as refinements of our informal conceptions), but we also recognise that carefully formulated theories can be used to correct our pre-theoretical conceptions. When it comes to the object-language of our logical theories we have already become accustomed to such splitting of notions. We recognise that with more than two truth-values negation and rejection may come apart, or that in the absence of weakening and contraction the binary connectives split into their intensional and extensional fragments.

What I propose is that a similar splitting of notions can also be at work at the level of our meta-theoretical concepts. We do, for instance, know from the relevantist tradition that in the absence of weakening and contraction the concept of logical consequence also splits into an internal and an external part. With this in mind, it would in fact be surprising if our thinking about meaning and content could do without further refinements. To repeat the moral of this paper once more: Pluralist theses are hard to formulate; it doesn't only take care, but also the right type of conceptual resources. But what counts as the "right type" of conceptual resources is an issue that cannot be judged independently from our formal resources.

As I've argued, many objections against logical pluralism do trade on the lack of such resources. In Hjortland's proposal, n-sided sequents provide the required formal resources to obtain an interesting type of logical pluralism. It only needs to be supplemented with an account of how meaning, content and synonymy function against the background of this formalism. The proposed divergence between meaning and content is hardly novel. It, does for instance, play a role that is similar to the distinction

[22]Contrast this with the related distinction between content and character in Kaplan's work (and to which Hjortland refers), where the relevant contrast is formalised but not motivated by Kaplan's logic of demonstratives.

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between the local and the global meaning of the connectives in substructural logic (e.g. Paoli 2003). Yet, in my defence of this distinction it is supported by (Hjortlands point) the inferentialist identification of the meaning of connectives with the rules of a proof-system; (my own contribution to the debate) the suggestion that we should avoid a mismatch between our conceptual resources and our formal resources; and (a general methodological recommendation) a plea for conceptual engineering (or for Carnap's logic-first approach) in the philosophy of logic.

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Appendix

Theorem 1 A and B are 3-synonymous iff $A \dashv\vdash_{\mathbf{LP}} B$ and $A \dashv\vdash_{\mathbf{K3}} B$.

Proof. $\Rightarrow$ If A and B are 3-synonymous, then (1) $A \mid A \mid B$, (2) $A \mid B \mid B$, (3) $B \mid B \mid A$, (4) and $B \mid A \mid A$ are derivable from the axiomatic sequent $A \mid A \mid A$. From (1) and (3) it follows that $A \dashv\vdash_{\mathbf{K3}} B$, while from (2) and (4) it follows that $A \dashv\vdash_{\mathbf{LP}} B$.
$\Leftarrow$ Assume that $\Gamma_0, C_1(A) \mid \Gamma_i, C_2(A) \mid \Gamma_1, C_3(A)$ is derivable and that A and B are both **LP** and **K3**-equivalent. If $\Gamma_0, C_1(A) \mid \Gamma_i, C_2(A) \mid \Gamma_1, C_3(A)$ is the final sequent of a cut-free proof π,[23] let $\pi^{A/B}$ be the result of replacing the final sequent in π by $\Gamma_0, C_1(B) \mid \Gamma_i, C_2(B) \mid \Gamma_1, C_3(B)$, and π^{A/B^*} the result of copying the actual replacements of $A's$ by $B's$ upward in the proof. A straightforward induction on the complexity of proofs suffices to establish that the leaves of π^{A/B^*} must be of one of the following types:

Type 1 $B \mid B \mid B$.
Type 2 $A \mid A \mid B$; $A \mid B \mid B$; $B \mid B \mid A$; or $B \mid A \mid A$
Type 3 $A \mid B \mid A$; or $B \mid A \mid B$.

Sequents of type 1 are axiomatic. Because A and B are, by assumption, **LP** and **K3**-equivalent, the sequents of type 2 should be derivable, whereas sequents of type 3 are derivable from sequents of type 2 by a single application of the cut-rule:

$$\frac{A \mid B \mid B \quad B \mid B \mid A}{A \mid B \mid A}\,(\mathrm{Cut}_{0,1}, B) \qquad \frac{B \mid A \mid A \quad A \mid A \mid B}{B \mid A \mid B}\,(\mathrm{Cut}_{0,1}, A)$$

This is all we need to extend π^{A/B^*} into a proof π' with final sequent $\Gamma_0, C_1(B) \mid \Gamma_i, C_2(B) \mid \Gamma_1, C_3(B)$, and only axiomatic sequents as its leaves. □

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[23] If a sequent is derivable, it is derivable without cut; see Theorem 3.9 of Baaz et al. (1993).

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